

Neutrino induced coherent pion production

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Abstract. We discuss different parameterizations of the $C_5^A(q^2)$ $N\Delta$ axial form factor, fitted to the old Argonne bubble chamber data for pion production by neutrinos, and we use coherent pion production to test their low q^2 behavior. We find moderate effects that will be difficult to observe with the accuracy of present experiments. We also discuss the use of the Rein-Sehgal model for low energy coherent pion production. By comparison to a microscopic calculation, we show the weaknesses of some of the approximations in that model that lead to very large cross sections as well as to the wrong shapes for differential ones. Finally we show that models based on the partial conservation of the axial current hypothesis are not fully reliable for differential cross sections that depend on the angle formed by the pion and the incident neutrino.

Keywords: Neutrino reactions, coherent pion production, Rein-Sehgal model, $N\Delta$ weak form factors

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INTRODUCTION

A proper understanding of coherent pion production is very important in the analysis of neutrino oscillation experiments since pion production is a source of background [1]. With pions being mainly produced through the excitation of nucleon resonances, coherent production can be used to extract information on axial form factors for the nucleon-to-resonance transition.

In coherent production the nucleus remains in its ground state and the reaction is controlled by the nucleus form factor. The nucleus form factor favors small values of the nucleus momentum transfer squared t . Small t values imply in this reaction small q^2 (square of the lepton momentum transfer). For small q^2 , coherent pion production is dominated by the divergence of the axial current and it can thus be related to the pion-nucleus coherent scattering through the partial conservation of the axial current (PCAC) hypothesis.

Experimental analyses of the coherent reaction rely on the Rein-Sehgal (RS) model [2] which is based on PCAC. In the RS model the pion-nucleus coherent cross section is written in terms of the pion-nucleon elastic cross section by means of approximations that are valid for high neutrino energies and small t and q^2 values. As pointed out in Refs. [3, 4], those approximations are not reliable for neutrino energies below/around 1 GeV and light nuclei, like carbon or oxygen.

There are other approaches to coherent production that do not rely on PCAC but on microscopic models for pion production at the nucleon level [3, 5]. The domi-

nant contribution to the elementary amplitude at low energies is given by the Δ -pole mechanism (Δ excitation and its subsequent decay into pion nucleon). Medium effects like the modification of the Δ mass and width in the medium, final pion distortion, evaluated by solving the Klein-Gordon equation for a pion-nucleus optical potential, as well as nonlocalities in the pion momentum, are very important and are taken into account in microscopic calculations. In the microscopic model of Ref. [6] background terms were included on top of the dominant Δ -pole contribution. The least known ingredients of the model are the axial $N\Delta$ transition form factors, of which C_5^A gives the largest contribution (See Eq. 1 of Ref. [7] for a form factor decomposition of the $N\Delta$ weak current). Besides, within the Adler model [8] used in Ref. [6], C_5^A determines all other axial form factors. This strongly suggested the readjustment of that form factor to the experimental data, which the authors did by fitting the flux-averaged $\nu_\mu p \rightarrow \mu^- p \pi^+$ Argonne (ANL) [9, 10] q^2 -differential cross section for pion-nucleon invariant masses $W < 1.4$ GeV, for which the model should be appropriate. They found $C_5^A(0) = 0.867$ which is some 30% smaller than the value predicted by the off-diagonal Goldberger-Treiman (GT) relation that predicts $C_5^A(0) = 1.2$. Background terms turn out to play a minor role in coherent production [3] where a reduced $C_5^A(0)$ value gives rise to smaller cross sections. Coherent production is dominated by the axial current which in microscopic models is, in its turn, dominated by the C_5^A axial form factor. Coherent production could then be used to test the validity of different $C_5^A(q^2)$ dependences in the small q^2

region.

In this contribution we will concentrate on the two issues mentioned before: first, we will try to see how sensitive coherent production is to different $C_5^A(q^2)$ parameterizations proposed in the literature. Second, we will present results that show how and why the RS model fails for low energy pion coherent production on light nuclei.

$C_5^A(q^2)$ IN COHERENT PRODUCTION

As mentioned before, in Ref. [6] the authors made a fit of $C_5^A(q^2)$ to ANL data. For C_5^A they took the q^2 dependence of Ref. [11]

$$C_5^A(q^2) = \frac{C_5^A(0)}{(1 - q^2/M_{A\Delta}^2)^2} \times \frac{1}{1 - \frac{q^2}{3M_{A\Delta}^2}}, \quad (1)$$

and from the fit they obtained

$$C_5^A(0) = 0.867 \pm 0.075, M_{A\Delta} = 0.985 \pm 0.082 \text{ GeV}. \quad (2)$$

This fitted axial mass in the weak $N\Delta$ vertex is in good agreement with the estimates of about 0.95 GeV and 0.84 GeV given in the original ANL reference [10] and in the work of Ref. [12]. On the other hand, a correction of the order of 30% to the off-diagonal GT relation value was found for $C_5^A(0)$. As shown in Ref. [6] the agreement with ANL total cross sections improved with the fitted values for $C_5^A(0)$ and $M_{A\Delta}$. On the other hand it is also shown that Brookhaven (BNL) bubble chamber data [13] and ANL data are not fully compatible and that BNL data alone would favor a $C_5^A(0) \approx 1.2$ value as given by the off-diagonal GT relation.

A different approach has been followed by Leitner *et al.* in Ref. [14]. There, the authors use a different parameterization for $C_5^A(q^2)$

$$C_5^A(q^2) = 1.2 \times (1 - \frac{aq^2}{b - q^2}) / (1 - q^2/M_{A\Delta}^2)^2, \quad (3)$$

in which $C_5^A(0)$ is kept to its off-diagonal GT relation value $C_5^A(0) = 1.2$, while a , b and $M_{A\Delta}$ are treated as free parameters. One can accommodate a larger $C_5^A(0)$ value by changing the q^2 dependence. In fact very small $-q^2$ values are not very relevant due to phase space and what is actually important is the $C_5^A(q^2)$ value in the region around $-q^2 \approx 0.1 \text{ GeV}^2$. When fitting the ANL data with the Δ -pole term alone they got [14]

$$a = -0.25, b = 0.04 \text{ GeV}^2, M_{A\Delta} = 0.95 \text{ GeV}. \quad (4)$$

But background terms are important at the nucleon level and they should be included in the calculation. A new fit

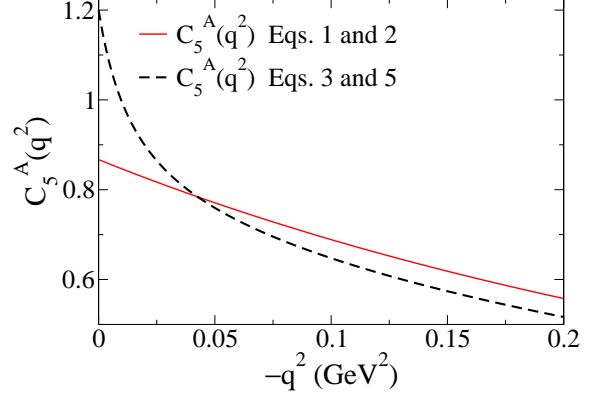


FIGURE 1. Different $C_5^A(q^2)$ parameterizations fitted to ANL data.

including background terms leads to

$$a = -0.3861 \pm 0.198, b = 0.01536 \pm 0.0310 \text{ GeV}^2, \\ M_{A\Delta} = 0.952 \pm 0.205 \text{ GeV}. \quad (5)$$

Both the fit in Eqs. 1 and 2 and the one in Eqs. 3 and 5 give a good description of ANL data even though they use quite different values for $C_5^A(0)$. The different q^2 dependence compensates for the initial difference at $q^2 = 0$. Note, however, the large errors in Eq. 5 that point at large correlations between the different parameters. In Fig. 1 we compare the two form factor parameterizations. The main differences are in the $-q^2 < 0.025 \text{ GeV}^2$ region. The corresponding axial radii are $R_A^2 = 0.56^{+0.10}_{-0.09} \text{ fm}^2$ for the parameterization given by Eqs. 1 and 2, and $R_A^2 = 6.4^{+0.9}_{-8.4} \text{ fm}^2$ for the one given by Eqs. 3 and 5. The large negative error in the latter case is a reflection of the large statistical errors in Eq. 5. Both things point to the fact that Eq. 3 might not be a good parameterization.

In the following we will try to see if coherent pion production can give us extra information on the validity of the two different $C_5^A(q^2)$ discussed above. Here we shall use the coherent production model of Ref. [3].

In Fig. 2 we show the q^2 differential cross section for charged current (CC) and neutral current (NC) coherent production on carbon and for an incident neutrino energy $E = 600 \text{ MeV}$. In both cases one can see differences for low $-q^2$ due to the different $C_5^A(q^2)$ form factor used. These effects are more relevant in the NC case where smaller $-q^2$ values are not suppressed by phase space. As $\frac{d\sigma}{dq^2}$ can not be measured for the NC channel one has to rely on total cross sections. There, the change is a mere 3.4% for the CC channel, while for the NC process, where lower $-q^2$ can be reached, the change is 16.6%.

In Fig. 3 we show the NC differential cross section with respect to $\cos \theta_\pi$, being θ_π the angle formed by the pion and the incident neutrino. In this case there is no difference in shape as can be appreciated by comparing

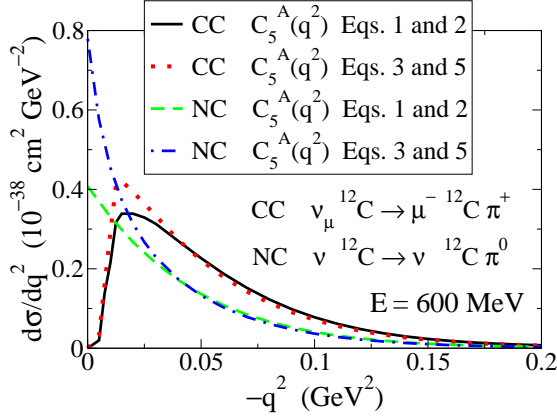


FIGURE 2. $\frac{d\sigma}{dq^2}$ differential cross sections for CC and NC neutrino induced one-pion production on carbon obtained with different $C_5^A(q^2)$ parameterizations. The incident neutrino energy is $E = 600$ MeV.

the solid line and the up triangle line. A similar result is obtained for the CC case and for the NC/CC differential cross section with respect to the pion kinetic energy.

Finally in Fig. 4 we show total cross sections. For the CC case a cut $|\vec{k}_\mu| > 450$ MeV on the final muon momentum, as the one imposed by the K2K Collaboration [15], has been applied. The changes are more significant for the NC case. In Table 1 we show the CC results convolved with the K2K flux and the NC ones convolved with the MiniBooNE flux. Details of the convolution can be found in Ref. [3]. We also show for comparison the experimental data. The change in the parameterization for $C_5^A(q^2)$ amounts to an small increase of 3% for the CC/K2K case. Both results are below the present experimental upper bound. For the NC/MiniBooNE case there is an increase of 10.7%, with the results of both parameterizations being a factor of 2 smaller than present experimental data [16].

Coherent pion productions shows a moderate dependence on different $C_5^A(q^2)$ parameterizations fitted to the ANL data. With present experimental accuracy it can not be used to constraint the form factor small q^2 behavior.

REIN-SEHGAL MODEL FOR LOW ENERGY COHERENT PRODUCTION

The RS model is based on PCAC, so it is worth motivating how one constructs a general PCAC based model. Let us look, for simplicity, at the NC process

$$\nu_l(k(E, \vec{k})) + \mathcal{N}_{gs} \rightarrow \nu_l(k'(E', \vec{k}')) + \mathcal{N}_{gs} + \pi^0(k_\pi). \quad (6)$$

Defining $q = k - k'$, $y = q^0/E$, $t = (q - k_\pi)^2$, taking \vec{q} along the positive Z axis and $\vec{k} \times \vec{k}'$ along the positive

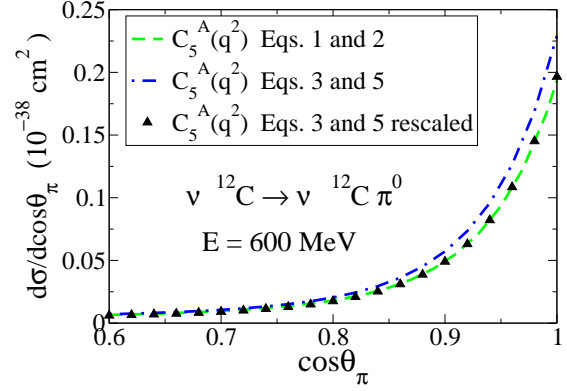


FIGURE 3. $\frac{d\sigma}{d\cos\theta_\pi}$ differential cross section for the $\nu {}^{12}\text{C} \rightarrow \nu {}^{12}\text{C} \pi^0$ reaction obtained with different $C_5^A(q^2)$ parameterizations. The incident neutrino energy is $E = 600$ MeV

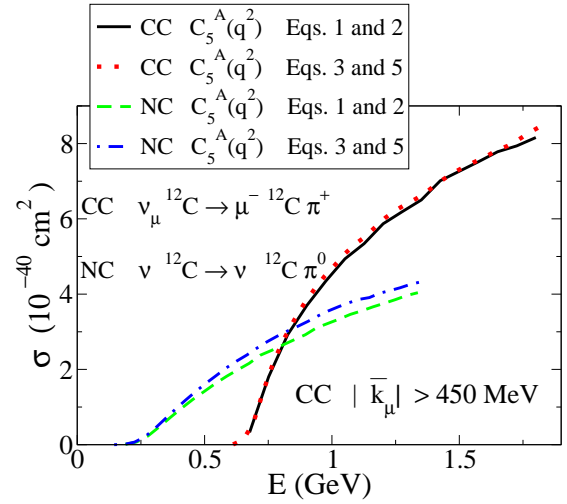


FIGURE 4. Total cross sections, as a function of the incident neutrino energy, for CC and NC neutrino induced one-pion production on carbon obtained with different $C_5^A(q^2)$ parameterizations. For the CC case a cut $|\vec{k}_\mu| > 450$ MeV on the final muon momentum has been applied.

Y axis, and calling $\phi_{\pi q}$ to the pion azimuthal angle in the XYZ frame, Lorentz invariance allows us to write

$$\frac{d\sigma}{dq^2 dy dt d\phi_{\pi q}} = \frac{G^2}{16\pi^2} \frac{-q^2 E \kappa}{|\vec{q}|^2} \times \left(\frac{u^2}{2\pi} \frac{d\sigma_L}{dt} + \frac{v^2}{2\pi} \frac{d\sigma_R}{dt} + 2 \frac{uv}{2\pi} \frac{d\sigma_S}{dt} + \frac{d\mathcal{A}}{dt d\phi_{\pi q}} \right), \quad (7)$$

where G is the Fermi decay constant, $\kappa = q^0 + \frac{q^2}{2M}$, with M the nucleus mass, and $u, v = \frac{E+E' \pm |\vec{q}|}{2E}$. Besides, $\sigma_{R,L,S}$ stand for cross sections for right, left and scalar polarized intermediate vector mesons. \mathcal{A} is not a proper cross section and it contains all the dependence on $\phi_{\pi q}$. As shown in Ref. [4], Eq. 7 should be the starting point to

TABLE 1. Total cross sections for CC and NC neutrino induced one-pion production on carbon convolved with the K2K and MiniBooNE fluxes. Different $C_S^A(q^2)$ parameterizations have been used. Experimental results are also shown.

Reaction	Experiment	$C_S^A(q^2)$	$\bar{\sigma}[10^{-40}\text{cm}^2]$	$\sigma_{\text{exp}}[10^{-40}\text{cm}^2]$
CC $\nu_\mu + {}^{12}\text{C}$	K2K	Eqs. 1 and 2	4.68	} < 7.7 [15]
CC $\nu_\mu + {}^{12}\text{C}$	K2K	Eqs. 3 and 5	4.82	
NC $\nu_\mu + {}^{12}\text{C}$	MiniBooNE	Eqs. 1 and 2	1.97	} $4.54 \pm 0.04 \pm 0.71$ [16]
NC $\nu_\mu + {}^{12}\text{C}$	MiniBooNE	Eqs. 3 and 5	2.18	

evaluate cross sections with respect to θ_π . PCAC based models take instead as a starting point

$$\frac{d\sigma}{dq^2 dy dt} = \frac{G^2}{16\pi^2} \frac{-q^2 E \kappa}{|\vec{q}|^2} \left(u^2 \frac{d\sigma_L}{dt} + v^2 \frac{d\sigma_R}{dt} + 2uv \frac{d\sigma_S}{dt} \right), \quad (8)$$

which is obtained from Eq. 7 after integration on $\phi_{\pi q}$, and they further assume

$$\frac{d\sigma}{dq^2 dy dt d\phi_{\pi q}} = \frac{1}{2\pi} \frac{d\sigma}{dq^2 dy dt}. \quad (9)$$

The latter is incorrect for $q^2 \neq 0$ and it will have consequences when evaluating cross sections with respect to variables that depend on θ_π .

For $q^2 = 0$ only σ_S contributes and one finds that $q^2 \frac{d\sigma_S}{dt} \Big|_{q^2=0}$ is given as the modulus square of the hadronic matrix element of the divergence of the weak current. Since the vector NC current is conserved one is left with the divergence of the axial current that can be related, through PCAC, to the pion-nucleon elastic cross section

$$q^2 \frac{d\sigma_S}{dt} \Big|_{q^2=0} = -4 \frac{k_\pi^0}{\kappa} f_\pi^2 \frac{d\sigma(\pi^0 \mathcal{N}_{gs} \rightarrow \pi^0 \mathcal{N}_{gs})}{dt} \Big|_{q^2=0}. \quad (10)$$

with $f_\pi = 92.4 \text{ MeV}$. Neglecting the nucleus recoil ($k_\pi^0 = q^0$) and including a form factor $G_A = 1/(1 - q^2/m_A^2)$ for $q^2 \neq 0$, one arrives at

$$\frac{d\sigma}{dq^2 dy dt} = \frac{G_F^2 f_\pi^2 Euv}{2\pi^2 |\vec{q}|} G_A^2 \frac{d\sigma(\pi^0 \mathcal{N}_{gs} \rightarrow \pi^0 \mathcal{N}_{gs})}{dt}. \quad (11)$$

This is the Berger-Sehgal model for π^0 coherent production [17]. In the RS model they further approximate

$$\frac{Euv}{|\vec{q}|} \rightarrow \frac{1-y}{y} \quad (\text{exact for } q^2 = 0), \quad (12)$$

$$\frac{d\sigma(\pi^0 \mathcal{N}_{gs} \rightarrow \pi^0 \mathcal{N}_{gs})}{dt} \rightarrow |F_A(t)|^2 F_{\text{abs}} \frac{d\sigma(\pi^0 N)}{dt} \Big|_{t=0}, \quad (13)$$

with $F_A(t)$ the nucleus form factor, F_{abs} a t -independent eikonal absorption factor that takes into account the distortion of the final pion, and $\frac{d\sigma(\pi^0 N)}{dt} \Big|_{t=0}$ the elastic pion-nucleon differential cross section at $t = 0$.

It is worth modifying the RS model by improving some of its approximations [4]. The $t = 0$ approximation can be eliminated altogether by substituting

$$\frac{d\sigma(\pi^0 N)}{dt} \Big|_{t=0} \longrightarrow \frac{d\sigma_{\text{nsp}}(\pi^0 N)}{dt}, \quad (14)$$

where nsp stands for the non-spin-flip part of the pion-nucleon elastic cross section. Besides, the pion distortion can be improved, still in an eikonal approach, with the replacement

$$|F_A(t)|^2 F_{\text{abs}} \longrightarrow \left| \int d^3\vec{r} e^{i(\vec{q}-\vec{k}_\pi)\cdot\vec{r}} \rho(\vec{r}) \Gamma(b, z) \right|^2, \\ \Gamma(b, z) = \exp \left(-\frac{1}{2} \sigma_{\text{inel}} \int_z^\infty dz' \rho(\sqrt{b^2 + z'^2}) \right). \quad (15)$$

Finally one can use $\frac{Euv}{|\vec{q}|}$ instead of $\frac{1-y}{y}$.

In Fig. 5 we evaluate $\frac{d\sigma}{dq^2}$ for NC coherent production on carbon at $E = 0.5 \text{ GeV}$. In order to check the $t = 0$ approximation of the RS model, no distortion is included in the calculation. We compare with the modified RS model where the $t = 0$ approximation has been removed. We also compare with the microscopic calculation of Ref. [3] that we consider a good model for coherent production at these energies. For simplicity, in the microscopic calculation we have only kept the dominant Δ contribution and, in order to make the comparison meaningful, we have taken $C_S^A(0) = 1.2$ (to fix normalization) and we have not included any in medium correction for the Δ or any pion distortion. We see that the modified RS model compares very well with the microscopic calculation, whereas the original RS model fails to reproduce both the size and the shape of the differential cross section.

In the same conditions as before, we show in Fig. 6 the $\frac{d\sigma}{d\eta}$ differential cross section on carbon at $E = 1 \text{ GeV}$. Once more, we see the size and shape of the RS calculation is very different from the microscopic one, showing the inadequacy of the $t = 0$ approximation. In this case the modified RS is also in disagreement with the microscopic calculation. This is mainly due to the approximation in any PCAC based model encoded in Eq. 9 and, to a

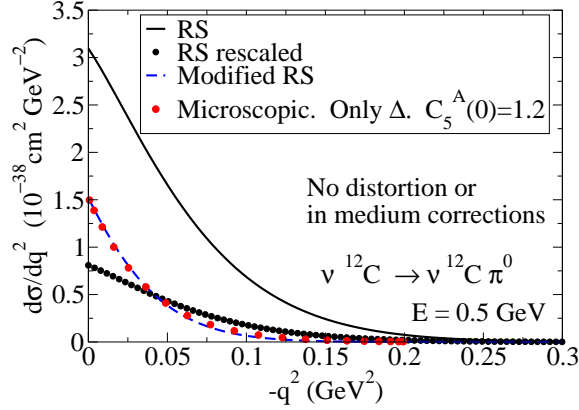


FIGURE 5. $\frac{d\sigma}{dq^2}$ differential cross sections for the $\nu^{12}\text{C} \rightarrow \nu^{12}\text{C} \pi^0$ reaction obtained within different models. The incident neutrino energy is $E = 0.5 \text{ GeV}$. No distortion or in medium corrections were considered in the calculations.

lesser extent, to the neglect by PCAC models of the $\sigma_{R,L}$ contributions. This assertion can be checked by eliminating from the microscopic calculation the non-PCAC contributions, i.e. taking only σ_S and assuming Eq. 9. The results are shown in Fig. 6 with up triangles. We now see the agreement with the modified RS calculation is good. This shows in fact that PCAC based models are not fully reliable to give differential cross sections with respect to variables that depend on θ_π .

Last, in Fig. 7 we present results for $\frac{d\sigma}{d\eta}$ but now with distortion and in medium corrections. Again, the sizes and shapes of the RS model predictions are very different from the microscopic ones. The same is true for the modified RS calculation, even when compared to the microscopic one with only PCAC contributions. While the shapes of the differential cross sections of the two latter calculations are similar, the integrated cross sections differ by a factor ≈ 1.7 . This hints at the inadequacy of eikonal factors to account for the final pion distortion or in medium modifications of the πN cross section. We see again that the approximations in PCAC models give rise to the wrong distributions for variables depending on θ_π .

As shown, the $t = 0$ approximation and the eikonal distortion used in the RS model are not adequate for low energy coherent production on light nuclei. Besides, and due to the assumption in Eq. 9, any PCAC model would predict the wrong shape for differential cross sections with respect to variables that depend on θ_π .

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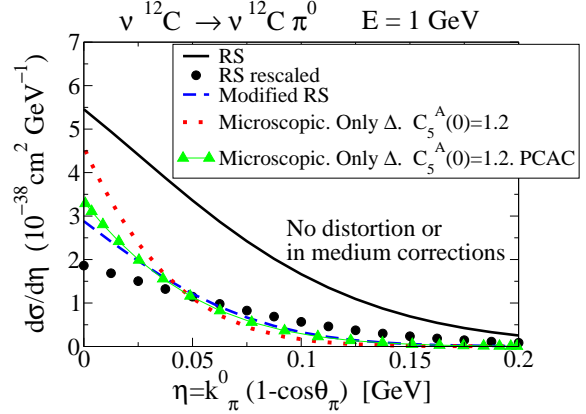


FIGURE 6. $\frac{d\sigma}{d\eta}$ differential cross sections for the $\nu^{12}\text{C} \rightarrow \nu^{12}\text{C} \pi^0$ reaction obtained within different models. The incident neutrino energy is $E = 1 \text{ GeV}$. No distortion or in medium corrections were considered in the calculations.

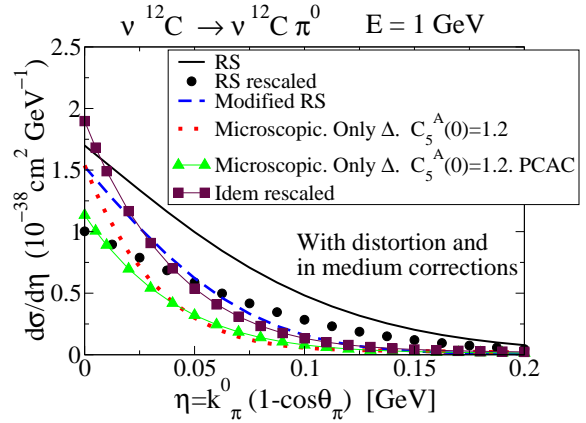


FIGURE 7. Same as Fig. 6, but now with pion distortion and in medium corrections taken into account.

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